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Abstract—This report presents a new digital redesign method for robust control of a sampled-data uncertain system using an observer-based digital controller. The multiple-segment trapezoidal rule together with interval arithmetic is utilized to find a digital interval model of the original continuous-time uncertain system. A dual concept of the digital interval modelling which captures the intersample states of the original continuous-time uncertain system is used to discretize a predesigned continuous-time state-feedback robust controller so that the states of the digitally controlled continuous-time uncertain system closely match those of the original analogously controlled continuous-time uncertain system. A discrete-time observer is constructed from the original continuous-time observer such that the estimated states of the redesigned discrete-time observer match those of the original continuous-time observer at the sampling instants. Using the newly digitally redesigned observer-based controllers, the resulting dynamic states of the digitally controlled sampled-data uncertain systems are able to closely match those of the original analogously controlled continuous-time uncertain systems.

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# Robust Optimal Digital Control of Uncertain Multi-Rate Sampled-Data Systems

# Summary of Research Results

Most physical processes and complex systems are formulated by hybrid uncertain systems that consist of mixed continuous and discrete uncertain subsystems. The uncertainties in these systems arise from unmodeled dynamics, parameter variations, sensor noises, actuator constraints, etc. These variations do not follow any of the known probability distributions in general, and are most often quantified in terms of amplitude and/or frequency bounds. Hence, practical systems are most suitably represented by hybrid interval models with bounded parameters, disturbances and noise inputs. At present, no effective method and software are available for digital modeling and digital simulation of such hybrid interval systems. With the aid of interval arithmetic, we have developed new interval methods for finding an equivalent discrete-time or continuous-time interval model for a hybrid interval system. As a result, the well-developed theories and methods in either the discrete-time domain or the continuous-time domain can be effectively applied to the analysis and design of hybrid uncertain systems.

Moreover, to improve significantly the qualitative and quantitative properties of an uncertain system represented by a continuous-time uncertain framework, analogue robust control design methods, such as the continuous-time H2/H∞ robust control methods, have been developed in the past. With the rapid advances in digital technology and computers, the resulting analogue robust controller is often required to be implemented using a digital controller for better reliability, lower cost, more flexibility and better performance. The process of converting an analogue controller to an equivalent digital controller, so that the performance of the digitally controlled hybrid uncertain system closely matches that of the original continuous-time controlled uncertain system for a relatively longer sampling period, is called "digital redesign". At present, no effective method is available for digital redesign of such hybrid uncertain systems. We have recently developed a new digital redesign method for robust digital control of continuous-time interval systems. The new digital controllers can be classified into two types: The PAM (Pulse-Amplitude Modulated) controller and the PWM (Pulse-Width Modulated) controller. The PAM controller, which produces a series of piecewise-constant continuous pulse having a variable amplitude and a fixed or variable width, is commonly used in digital control systems of all types, while the

PWM controller, which provides a series of discontinuous pulse with a fixed amplitude and a variable width, has become popular in military and industry for on-off control of DC power converters, stepper motors widely used in robotics, satellite station-keeping (with on-off rejection jets), etc. Utilizing the practically implementable digital controllers, the resulting dynamic states of the digitally controlled sampled-data interval systems are able to closely match those of the original analogously controlled continuous-time interval systems.

Also, the PAM and PWM controllers newly developed us have been successfully implemented and tested on the rigid and flexible body dynamics of the Hubble Space Telescope extended on the Shuttle's Remote Manipulator by using the high-fidelity non-linear Interactive On-orbit Simulation/Draper RMS Simulation (IOS/DRS). The details of the development and simulation results have been presented at the AIAA Guidance, Navigation and Control Conference held in Baltimore, August 1995, and the SPIE's Symposium on Mathematics and Control in Smart Structures held in San Diego, February 1996.

Based on the research results in the period of June 1,1994 to January 31,1998, twenty-eight technical papers have been published in the referred journals and listed as follows:

(1). Wei, C.P., J.S. Tsai, and L.S. Shieh, "Design of Optimal Digital Servo Controllers for Continuous-time Input-delay Systems," Journal of Control Systems and Technology, Vol. 1, No. 4, pp. 267-283 1993.

#### **Abstract**

This paper deals with the design method of an optimal robust digital servo controller with two degrees of freedom for a continuous-time plant with an input delay. The proposed digital controller includes not only state-feedback terms from the plant and the servocompensator but also state-feedforward terms from the reference and disturbance signals. We introduce a discrete-time performance index to optimize the feedback and feedforward gains and use state reconstructors to realize the servo controller. As a result, the sampling output of the digital system tracks the sampling reference signal in the presence of the disturbance. In addition, the use of the optimal feedforward compensation leads to a satisfactory transient response to the reference signal.

(2). Tsai, J.S.H., W.S. Chen, L.S. Shieh, "Realization, Design and Implementation of Row-Pseudoproper Left Matrix Fraction Descriptions with Impulsive Modes," Computers Math. Applic., Vol. 28, No. 7, pp. 13-41, 1994.

## **Abstract**

A novel and minimal realization algorithm is proposed for determining generalized state-space representation from a so-called row-pseudoproper left matrix fraction description (LMFD). The realized system with the state-space representation form is proved to be controllable and observable in the sense of [1,2] if the given row-pseudoproper MFD is left coprime. Besides, the proposed state feedback control law not only satisfies the optimal regional-pole-placement design for the realized generalized dynamical system, but also eliminates the impulsive terms in the state response of the closed-loop system. For practical consideration, an equivalent input-output feedback structure of the designed state-feedback controller is adopted. Based on the cascaded and/or parallel active RC networks with better sensitivity and stability properties, the resulting structure of the equivalent input-output feedback controller can be readily implemented based on proper subsystems.

(3). Tsai, J.S.H., C.P. Wei and L.S. Shieh, "Design of a Stable Digital Output Feedback Controller for Continuous-time Input-delay Plants," International Journal of Systems Science Vol. 25, No. 12, pp. 2187-2206, 1994.

## **Abstract**

This paper presents a stable multirate output-feedback controller for digital control of a continuos-time input-delay system. It is shown that an arbitrary digital state feedback controller for an observable input-time-delay plant can be realized by using multirate sampled output data. The developed digital multirate-output feedback controller enables us to achieve results similar to those of the state feedback controller, without constructing an observer.

(4). Tsai, J.S.H., F.C. Liu and L.S. Shieh, "Model Conversions of Uncertain Linear Time-delay Systems Using the Block-pulse Function Approach," Journal of Control Systems and Technology, Vol. 2, No. 2, pp. 127-136, 1994.

#### Abstract

This paper proposes a block-pulse function approach along with the digital redesign concept to obtain the model conversions of a continuous-time (discrete-time) uncertain linear time-delay system to an equivalent discrete-time (continuous-time) uncertain linear time-delay model. The proposed method allows the use of well-developed theorems and algorithms in the discrete-time (continuous-time) domain to indirectly solve continuous-time (discrete-time) domain problems. The continuous-time uncertain time-delay system is considered to be composed of continuous-time nominal matrices with uncertainties and an input part with pure delay time. The digital redesign concept is used to determine the discrete-time uncertainties from the continuous-time uncertain time-delay system, and vice versa. Moreover, the paper also proposes a method to estimate tighter bounds of discrete-time (continuous-time) structured uncertainties based on the given continuous-time (discrete-time) structured uncertainties. An example is given to demonstrate the effectiveness of the proposed method.

(5). Wang, S.G. and L.S. Shieh, "Robustness of Linear Quadratic Regulators with Regional-Pole Constraints for Uncertain Linear Systems," Control-Theory and Advanced Technology, Vol. 10, No. 4, Part 1, pp. 737-769, 1994.

#### Abstract

This Paper presents robust pole clustering bounds for uncertain linear systems with nominal linear quadratic regulators (LQRs). The pole clustering regions of interest are the vertical strip, the horizontal strip and the ring (including the circle as a special case) with a center at the real axis in the s-plane. The Raleigh principle, along with the conventional norm theory, instead of the commonly used Lyapunov-type approach, is utilized for determining the robustness bounds of the poles of the perturbed closed-loop systems. Also, a constraint is established to verify whether the proposed nominal LQRs with robust pole clustering still preserve optimality with respect to a specific quadratic cost function. Comparison of our method with some other existing methods is carried out using illustrative examples.

(6). Wang, S.G. and L.S. Shieh, "Robust Optimal Pole-Clustering in a Vertical Strip and Disturbance Rejection for Uncertain Lagrange's Systems," Dynamics and Control, Vol. 5, pp. 295-312, 1995.

#### Abstract

This paper presents an approach to design a state-feedback robust control law for uncertain Lagrange's systems such that the designed closed-loop systems have the properties of robust pole-clustering within a vertical strip and disturbance rejection with an H∞-norm constraint. This approach is based on solving an algebraic Riccati equation with the adjustable scalars and prespecified parameters. The uncertainties considered include both unstructured and structured uncertainties in the system and the input matrices. Also, a constraint is established to verify that the proposed robust LQRs preserve H2 optimality with respect to a specific quadratic cost function.

(7). Wang, S.G., L.S. Shieh and J. W. Sunkel, "Robust optimal pole-placement in a vertical strip and disturbance rejection," Int. J. Systems Sci., Vol. 26, No. 10, pp. 1839-1853, 1995.

#### Abstract

This paper presents a linear quadratic regulator (LQR) for robust closed-loop poleplacement within a vertical strip, and disturbance rejection with an H∞-norm constraint for the uncertain linear systems. The concerned systems cover both matched and mismatched uncertain linear systems with unstructured or structured uncertainties existing in both the system and input matrices. A set of tuning parameters is incorporated for some flexibility in finding a solution to the algebraic Riccati equation, and a controller dain parameter is selected for robust pole clustering. Also, a constraint is established to verify whether the proposed robust LQRs preserve H2 optimality with respect to a specific quadratic cost function.

(8). Shieh, L.S., X. Zou and N.P. Coleman, "Digital interval model conversion and simulation of continuous-time uncertain systems," IEE Proceedings D. Control Theory and Applications,, Vol. 142, No. 4, pp. 315-322, 1995.

#### Abstract

The paper deals with the problem of converting a continuous-time uncertain linear system to an equivalent discrete-time uncertain model and its digital simulation. The system matrices characterising the state-space representation of the original uncertain system are

assumed to be interval matrices. The geometric series method together with interval arithmetic is employed to obtain the approximate discrete-time interval models. A new technique is developed to estimate the modelling errors. These modeling errors are used to modify the approximate interval models obtained via the interval geometric-series method. The resulting interval models (the enclosing interval models) are able to tightly enclose the exact uncertain model. Also their approximate discrete-time interval solutions are able to tightly enclose the exact interval solution of the continuous-time uncertain state-space equation. The proposed digital uncertain models can be used for digital simulation and digital design of continuous-time uncertain systems.

(9). Shieh, L.S., J. Gu and J.S.H. Tsai, "Model conversions of uncertain linear systems using a scaling and squaring geometric series method," Circuits Systems Signal Process, Vol. 14, No. 4, pp. 445-463, 1995.

## **Abstract**

This paper proposes a scaling and squaring geometric series method along with the inverse-geometric series method for finding discrete-time (continuous-time) structured uncertain linear models from continuous-time (discrete-time) structured uncertain linear systems. The above methods allow the use of well-developed theorems and algorithms in the discrete-time (continuous-time) domain to indirectly solve the continuous-time (discrete-time) domain problems. Moreover, these methods enhance the flexibility in modeling and control of a hybrid composite system. It has been shown that the commonly used bilinear approximation model is a specific class of the proposed geometric series model.

(10). Tsai, J.S.H., C.P. Fan and L.S. Shieh, "Implementation of State-feedback Control Law for Singular Systems Via an Input-Output Feedback Structure," International Journal of Systems Science, Vol. 26, No. 11, pp. 2139-2158, November, 1995.

#### Abstract

The paper presents a state-feedback design methodology for singular systems and proposes an input-output feedback structure for implementation of the designed state-feedback control law. First, the state-space representation of a singular system is decomposed into a reduced-order regular subsystem and a fast subsystem which include impulsive modes and infinite nondynamic modes, in which the states of impulsive modes cannot be estimated by

means of the conventional state observer. Next, a state-feedback design concept is introduced to eliminate all impulsive modes of the fast subsystem and to optimally place the eigenvalues of the regular subsystem within a specific region. Finally, the obtained state-feedback control law is implemented by the proposed stable input-output feedback structure.

(11). Shieh, L.S., J. Gu and J.S.H. Tsai, "Robust Digital Redesign of Uncertain Linear Systems Using the Interval Bilinear Approximation Method," Control Theory and Advanced Technology, Vol. 10, No. 4, Part 4, pp. 1667-1688, November, 1995.

# **Abstract**

This paper presents an interval bilinear approximation method to convert a continuous-time state-feedback robust control law to an equivalent discrete-time counterpart for robust digital control of a continuous-time uncertain system. For performing the digital redesign of a continuous-time uncertain system, the given continuous-time uncertain system is converted to a tightly bounded equivalent discrete-time uncertain model using the bilinear approximation method aided by the interval arithmetic operations. Also, utilizing the interval bilinear approximation method, the continuous-time robust control law is converted to an equivalent discrete-time interval control law. Applying this digitally redesigned control law, the states of the digitally controlled sampled-data uncertain system closely match those of the original continuous-time controlled uncertain system. A numerical example is presented to demonstrate the effectiveness of the proposed method.

(12). Shieh, L.S., I.C. Lin and J.S.H. Tsai, "Design of PWM Controller for Sampled-Data System Using Digitally Redesigned PAM controller," Proceedings of IEE, Part D. Control Theory and Applications, Vol. 142, No. 6, pp. 654-660, November, 1995.

#### Abstract

Two issues are addressed: Digital redesign of a continuous-time system with an input time delay, and translation of the pulse-amplitude modulated (PAM) controller obtained by a newly proposed digital redesign method into an equivalent pulse-width modulated (PWM) controller. A tuning parameter is introduced into the PAM controller so that the digitally controlled sampled-data states closely match the original continuous-time input time-delay states. Also, the principle of equivalent areas is applied to convert the newly developed

PAM controller into an equivalent PWM controller so that the PWM controlled states closely match the PAM controlled states for a relatively longer sampling period. Two illustrative examples are provided to demonstrate the effectiveness of the proposed method.

(13). Shieh, L.S., W.M. Wang, and J.S.H. Tsai, "Digital Modelling and Digital Redesign of Sampled-Data Uncertain Systems," Proceedings of IEE, Part D., Control Theory and Applications, Vol. 142, No. 6, pp. 585-594, November, 1995.

## **Abstract**

A new method for digital model conversion of a continuous-time uncertain system and a new digital redesign method for robust control of a sampled-data uncertain system are presented. The concept of the principle of equivalent areas together with interval arithmetic is utilised for finding the discrete-time uncertain model and the digital robust control law from the continuous-time uncertain state equation and the analogue robust control law, respectively. Using the newly digitally redesigned controllers, the resulting dynamic states of the digitally controlled sampled-data uncertain systems are able to closely match those of the original analogously controlled continuous-time uncertain systems for a relatively longer sampling period.

(14) Tsai, J.S.H., F.C. Liu and L.S. Shieh, "Model Conversions of Uncertain Linear Time-Delay Systems Using a Scaling and Squaring Geometric Series Method," Control Theory and Advanced Technology, Vol. 10, No. 4, Part 5, pp. 2145-2171, December 1995.

# **Abstract**

This paper proposes the scaling and squaring geometric series method for the model conversion of a continuous-time (discrete-time) uncertain linear time-delay system to an equivalent discrete-time (continuous-time) uncertain linear time-delay model. The proposed method allows the use of well-developed theorems and algorithms in the discrete-time (continuous-time) domain to solve indirectly the continuous-time (discrete-time) domain problems. The system and input matrices characterizing the state-space representation of the original linear uncertain system with time-delay inputs are assumed to be uncertain matrices. The geometric-series and inverse-geometric-series approximation methods are used to determine the discrete-time uncertain time-delay model from the continuous-time

uncertain time-delay system and vice versa. An example is given to demonstrate the effectiveness of the proposed method.

(15) Shieh, L.S., R.K. Kasavaraju and J.S.H. Tsai, "Digital Redesign of Continuous-Time Controller Using Pade and Inverse-Pade Approximation Method," Journal of the Franklin Institute, Vol. 332B, No. 4, pp. 433-442, 1995.

#### **Abstract**

This paper presents a mixed method consisting of the Pade approximation method and the inverse-Pade approximation method for digital redesign of a continuous-time state-feedback controller. Using this digitally redesigned controller, the states of digitally controlled sampled-data system closely match those of the original continuous-time controlled system for a relatively longer sampling period. It is shown that the digitally redesigned controller based on the bilinear and inverse-bilinear transform method is a specific class of the proposed controller.

(16). Shieh, L.S., X. Zou and N.P. Coleman, "Digital Interval Modelling and Hybrid Control of Uncertain Systems," IEEE Trans. Industrial Electronics, Vol. 43, No. 1, pp. 173-183, February, 1996.

#### Abstract

This paper addresses two issues: 1) converting a continuous-time uncertain system to an equivalent discrete-time interval model; and 2) constructing a robust digital control law from a robust analogue control law for hybrid control of sampled-data uncertain systems. The system matrices characterizing the state-space representation of the original continuous-time uncertain systems are assumed to be interval matrices. The Pade approximation method together with a geometric-series approximation method is employed to obtain the generalized enclosing discrete-time interval models. The generalized enclosing interval models are able to tightly enclose the exact discrete-time uncertain model, and can be utilized for digital simulation and digital design of continuous-time uncertain systems.

A new family of digitally redesigned interval controllers is constructed from a continuoustime robust controller for robust digital control of continuous-time uncertain systems. Using the newly digitally redesigned interval controllers, the dynamic states of the digitally controlled sampled-data uncertain systems are able to closely match those of the original analogously controlled continuous-time uncertain systems for a relatively longer sampling period.

(17). Shieh, L.S., J. Gu and J.S.H. Tsai, "Model Conversions of Uncertain Linear Systems Via the Interval Pade Approximation Method," Circuits, Systems, and Signal Processing, Vol. 15, No. 1, pp. 1-22, 1996.

## **Abstract**

This paper presents a new interval Pade approximation method to convert a continuous-time (discrete-time) uncertain linear system to an equivalent discrete-time (continuous-time) uncertain model via interval arithmetic operations. Based on the inclusion theorem related to the interval arithmetic, the interval Pade's approximants and their associated interval error matrices with interval arguments are obtained via the Pade's approximants and their associated error matrices with degenerate (real) arguments, respectively. Tighter error bounds of various approximate uncertain models with respect to their exact uncertain models are determined and used to modify the obtained Pade's approximants, so that the resulting approximate uncertain models are able to tightly enclose the original uncertain systems. Thus, the analysis and design of the original uncertain systems can be indirectly carried out using the converted uncertain models in either the continuous-time or the discrete-time domain.

(18) Shieh, L.S., X. Zou and J.S.H. Tsai, "Model Conversions of Continuous-Time Uncertain Systems Via the Interval Geometric-Series Method," IEEE Trans. Circuits and Systems, Vol. 43, No. 10, pp. 851-854, October 1996.

# **Abstract**

This brief presents an interval geometric-series approximation method to covert a continuous-time uncertain system to an equivalent discrete-time uncertain model. The system matrices characterizing the state-space representation of the original uncertain systems are assumed to be interval matrices. The exponential matrix-valued function with an interval system matrix is approximated by a rational interval matrix-valued function using the geometric-series approximation method. Then, the desired enclosing interval approximation is obtained by adding an error interval matrix, which accounts for the approximation error, to the rational interval approximant. The model thus constructed is

guaranteed to enclose the precise original interval model. The proposed enclosing digital interval model provides less conservative results than the existing Pade enclosing digital interval model. The newly developed digital interval model can be utilized for analysis and design of continuous-time uncertain systems.

(19) Xu, J., G. Chen and L.S. Shieh, "Digital Redesign for Controlling the Chaotic Chua's Circuit," IEEE Trans. Aerospace and Electronics Systems, Vol. 32, No. 4, pp. 1488-1499, October 1996.

#### Abstract

We apply some successful digital redesign techniques, developed previously for the control of linear systems, to controlling the nonlinear chaotic Chua's circuit. Chua's circuit is a simple autonomous physical device that exhibits very rich and complex nonlinear dynamics of bifurcation and chaos, and is hence very sensitive to digital controls. To apply advanced high-speed computer technology to the implementation, we show how to redesign a good digital controller, based on an existing successful analog controller, for controlling the chaotic trajectories of Chua's circuit, from anywhere within the chaotic attractor to a predesired unstable limit cycle of the circuit.

(20) Shieh, L.S., W.M. Wang and J.W. Sunkel, "Design of PAM and PWM Controllers for Sampled-Data Interval Systems," ASME Trans. Journal of Dynamic Systems, Measurement, and Control, Vol. 118, pp. 673-682, December 1996.

#### Abstract

In this paper, we address two issues: digital redesign of a continuous-time interval system using an interval chebyshev quadrature approximation method; and translation of the newly digitally redesigned pulse-amplitude modulated (PAM) controller into an equivalent pulsewidth modulated (PWM) controller via a second-order Taylor-series approximation method. Using this new interval digital redesign technique, the dynamic states of the digitally controlled sampled-data interval system are able to closely match those of the original analogously controlled continuous-time uncertain system. Three illustrative examples are provided to demonstrate the effectiveness of the proposed methods.

(21) Xu, J., G. Chen and L.S. Shieh, "Digital Redesign for Controlling the Chaotic Chua's Circuit," IEEE Trans. Aerospace and Electronics System, Vol. 32, No. 4, pp.1488-1499, October 1996.

#### Abstract

We apply some successful digital redesign techniques, developed previously for the control of linear systems, to controlling the nonlinear chaotic Chua's circuit. Chua's circuit is a simple autonomous physical device that exhibits very rich and complex nonlinear dynamics of bifurcation and chaos, and is hence very sensitive to digital controls. To apply advanced high-speed computer technology to the implementation, we show how to redesign a good digital controller, based on an existing successful analog controller, for controlling the chaotic trajectories of Chua's circuit, from anywhere within the chaotic attractor to a predesired unstable limit cycle of the circuit.

(22) Shieh, L.S., W.M. Wang and J.W. Sunkel, "Digital Redesign of Cascaded Analogue Controllers for Sampled-Data Interval Systems," Proceedings of IEE, Part D., Control Theory and Applications, Vol. 143, No. 6, November 1996.

#### Abstract

The paper presents a new method for the digital redesign of cascaded analogue controllers for continuous-time parametric interval systems. The bilinear and inverse-bilinear approximation method is developed to carry out discretisation of the predesigned cascaded analogue controller, taking into account intersample behaviour and implementation errors. A dual concept of the proposed digital redesign method is utilised to construct a new pseudodigital observer such that the estimated states of the digitally redesigned digital observer closely match those of the original continuous-time observer at the sampling instants. Using the newly digitally redesigned observer based controllers, the resulting dynamic states of the digitally controlled cascaded sampled-data interval systems are able to closely match those of the original analogously controlled cascaded continuous-time parametric interval systems.

(23) Shieh, L.S., W.M. Wang and J.B. Zheng, "Robust Control of Sampled-Data Uncertain Systems Using Digitally Redesigned Observer-Based Controllers," International Journal of Control, Vol. 66, No.1, pp. 43-64, January 1997.

#### **Abstract**

This paper presents a new digital redesign method for robust control of a sampled-data uncertain system using an observer-based digital controller. The multiple-segment trapezoidal rule together with interval arithmetic is utilized to find a digital interval model of the original continuous-time uncertain system. A dual concept of the digital interval modelling, which captures the intersample states of the original continuous-time uncertain system is used to discretize a predesigned continuous-time state-feedback robust controller so that the states of the digitally controlled continuous-time uncertain system closely match those of the original analogously controlled continuous-time uncertain system. A discrete-time observer is constructed from the original continuous-time observer such that the estimated states of the redesigned discrete-time observer match those of the original continuous-time observer at the sampling instants. Using the newly digitally redesigned observer-based controllers, the resulting dynamic states of the digitally controlled sampled-data uncertain systems are able to match closely those of the original analogously controlled continuous-time uncertain systems.

(24) Chen, G., J. Wang and L.S. Shieh, "Interval Kalman Filtering," IEEE Trans. Aerospace and Electronics Systems, Vol. 33, No. 1, pp. 250-259, January 1977.

# Abstract

The classical Kalman filtering technique is extended to interval linear systems with the same statistical assumptions on noise, for which the classical technique is no longer applicable. Necessary interval analysis, particularly the notion of interval expectation, is reviewed and introduced. The interval Kalman filter (IKF) is then derived, which has the same structure as the classical algorithm, using no additional analysis or computation from such as H<sup>∞</sup>-mathematics. A suboptimal IKF is suggested next, for the purposed of real-time implementation. Finally, computer simulations are shown to compare the new interval Kalman filtering algorithm with the classical Kalman filtering scheme and some other existing robust Kalman filtering methods.

(25) Tsai, J.S.H., D.H.Li and L.S.Shieh, "Model Conversion of Uncertain Linear Systems with Input Time-Delay via the Interval Bilinear Approximation Method," Journal of Franklin Institute, Vol. 334B, No.1, pp. 23-40, 1997.

# Abstract

This paper presents a method for model conversion of a continuous-time (discrete-time) uncertain linear system with input time-delay to an equivalent discrete-time (continuous-time) uncertain linear time-delay model. The bilinear approximation method and interval arithmetic operations are proposed for converting continuous-time (discrete-time) uncertain input time-delay models to equivalent discrete-time (continuous-time) uncertain input time-delay models. Here, we also present a method to estimate tighter error bounds of discrete-time (continuous-time) structured uncertainties based on the given continuous-time (discrete-time) structures uncertainties. By applying the above proposed methods, the converted model tightly encloses the original system; consequently, the analysis and design can be carried out using the obtained models instead of the original systems. An example is given to demonstrate the effectiveness of the proposed method.

(26) Siouris, G.M., G. Chen and J.R. Wang, "Tracking an Incoming Ballistic Missile Using an Extended Interval Kalman Filter," IEEE Trans. Aerospace and Electronic Systems, Vol. 33, No. 1, pp. 232-240, January 1997.

#### Abstract

The important tracking problem by radar of an incoming ballistic missile system, which contains uncertainty in modeling and noise in both dynamics and measurements, is studied. The classical extended Kalman filter (EKF) is no longer applicable to such an uncertain system, and so a new extended interval Kalman filter (EIKF) is developed for tracking the missile system. Computer simulation is presented to show the effectiveness of the EIKF algorithm for this uncertain and nonlinear ballistic missile tracking problem.

(27).Feng,F.F,L.S.Shieh and G.Chen, "Model Conversions of Uncertain Linear Systems Using Multi-Point Pade Approximation" Applied Mathematical Modelling, Vol.21,No.4,pp.233-244,1997.

Abstract Many dynamic systems and industrial control processes can be represented by a multi-rate sampled-data uncertain system, which consists of a continuous-time uncertain subsystem and a multi-rate discrete-time uncertain subsystem. The uncertainties in these systems arise from unmodeled dynamics, parameter variations, sensor noises, actuator constraints, etc. As is the common practice, the sampled-data uncertain system needs to be converted to a purely continuous-time or discrete-time uncertain model, so that the well-established analysis and design methods in the continuous-time or discrete-time domain

can be directly applied to the equivalent model. This paper presents a new interval multipoint Pade approximation method for converting a continuous-time (discrete-time) uncertain linear system to an equivalent discrete-time (continuous-time) uncertain model. The system matrices characterizing the state-space descriptions of the original uncertain systems are represented by interval matrices. Using the approximate uncertain models obtained based on interval analysis and multipoint Pade approximation, the dynamic states of the resulting models have shown to be able to closely match those of the original uncertain systems for a relatively longer sampling period.

(28).Shieh,L.S.,J.B.Zheng and W.M.Wang, "Digital Modeling and Digital Redesign of Analog Uncertain Systems Using Genetic Algorithms," AIAA fournal of Guidance Control and Dynamics, Vol. 20, No. 4, pp721-728, 1997.

Abstract — This paper utilizes genetic algorithms to find the equivalent discrete-time uncertain model of a continuous-time uncertain system for digital simulation and digital design of the continuous-time uncertain system. The developed digital interval model provides less conservative results than those obtained by the conventional bilinear transform method. Also, the global optimization searching technique provided in genetic algorithms is used to determine the digital control law, taking into account the intersample behavior and implementation errors, for digital control of continuous-time parametric uncertain systems. The developed digitally redesigned control law is able to optimally match the states of the analogously controlled uncertain system and those of the digitally controlled sampled-data uncertain system. Moreover, it produces less conservative results than those obtained by the existing interval method. An illustrative example is included to demonstrate the proposed method.

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# Scientific Personnel

S.G. Wang (Ph.D, 94), R.K. Kasvaraju (MS, 94), F.F. Feng (MS, 96), U. Dikkala (MS, 96), A.C. Tsai (MS, 97), W.M. Wang (MS, 98), X. Zou (Ph.D. 97).